Project Title: **OPTIMAL ROOM AND PILLAR MINE SEQUENCING FOR INCREASED PROFITABILITY AND REDUCED RISK**

ICCI Project Number: 13/3B-1  
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**ABSTRACT**

Room-and-pillar (R&P) mining is the most common method used to produce coal from underground mines. This project addressed two key challenges faced by mine engineers involved in the design and operation of R&P coal mines. Those challenges are determining the optimal number of entries to extract in panels and determining the optimal production/extraction sequence for mining. Thus, the two-fold objective of this work was to: (i) evaluate the impact of variable panel width on unit cost and productivity of R&P operations using discrete event simulation (DES); and (ii) develop a mixed-integer linear programming model that incorporates risk management in determining the optimal extraction sequence for R&P panels. This research was carried out with the support of an Illinois-based R&P coal mine.

To achieve these objectives, the research team built a valid DES model of R&P mining, which was used in conducting simulation experiments to determine the optimal panel width based on unit costs and productivity. The model was validated using time-study data collected over two consecutive days for eighteen cuts. For this particular data set, the deviation between model and time-study data was 11% for the number of loads mined and 8% for the duration of mining. Thirty-six simulation experiments were conducted by varying the number of entries for advance, the number of additional rooms, and the number of haulage units assigned to each continuous miner.

For the particular operating conditions evaluated in this study, there exists an optimal panel width that maximizes productivity and unit costs. That width is based on certain objectives specified by the mine operator. Altering objectives supplied to the modeling exercise may result in a different answer. Fixed costs are a significant factor in deciding the optimal panel width based on unit costs. For the cooperating mine, developing R&P panels with 11-entry advance and three rooms on each side is recommended based on productivity. While mining in this configuration, four haulage units should be assigned to each continuous miner.
EXECUTIVE SUMMARY

Background

Room-and-pillar (R&P) mining is used by 90% of underground mines in the United States of America (USA) to extract flat and tabular deposits. Over 60% of coal produced in the USA comes from R&P mining (Tien, 2011). Mine planning and design is an important part of managing R&P mines. Some of the major challenges inherent in using R&P methods include risk associated geotechnical challenges, grade control, and recovery. The need to design safe mines with competent roof structures results in most mine designs ignoring the effect of design parameters such as panel width. The original scope of this project was to incorporate mining risk into mixed integer linear programming models of optimal mining sequences as well as investigate the effect of pillar recovery on net present value (NPV). The scope was modified at the request of the cooperating mine to include an evaluation of panel width with the objective of determining an optimal width based on cost and productivity.

Two of the most important challenges of R&P mine planning and design are: (i) how to determine the optimal number of entries to use in designing and producing from panels; and (ii) how to determine the optimal production/extraction sequence for mining panels. Panel design affects coal recovery and mining sequence, which in turn affects mining costs and productivity. To the best of the author’s knowledge no previous work has studied, in this detail, the effect of panel width (measured by number of entries) on productivity and unit costs. Most panel width design decisions are based only on past experience. This project sought to fill the gap between heuristic and empirical decision making with regard to panel width and extraction sequence.

Modeling R&P mining includes simulating several processes in the production cycle that adhere to very strict sequencing requirements (Newman and Kuchta, 2010) and managing mining risks (such as those associated with quality, production, and geotechnical conditions). Although, several different methods have been introduced for modeling and solving R&P sequencing problems in the literature, these approaches do not account for managing multiple risks. Common commercial software used for production sequencing in R&P operations use heuristic methods that only find approximate solutions to production sequencing problems. This project is an attempt to develop algorithms to control mining risks and their effect on optimal production sequences.

Objectives

Project objectives were to:

- Evaluate the impact of variable panel width on the unit cost and productivity of R&P operations using discrete event simulation (DES); and
- Develop a mixed-integer linear programming (MILP) model that incorporates risk management in determining the optimal sequence of panel extraction.
Experimental Procedures

The first objective was achieved by building a valid discrete event simulation (DES) model of R&P mining. Simulation experiments were designed to assess the effect of panel width (number of entries) on unit mining cost and productivity. For the particular case being examined in this study, experiments evaluated whether to advance the panel with 11 or 13 entries before mining rooms; and how many rooms (i.e., 0, 1, ..., 5) to mine on each side. Experiments were also conducted to assess the sensitivity of results to the number of haulage units assigned to each continuous miner. For each experimental set-up, 150 replications were run with each replication continued until all mining faces in the sequence had been completed (i.e., full width of the panel is mined out). A cut sequence was provided as an input based on mining practices at the cooperating mine.

The second objective was achieved by modeling R&P mine sequencing as a MILP problem. The model was formulated to maximize NPV while managing risk. The model includes resources, precedence, mining rate, reserve quantity and quality, and block-in-panel constraints. The research team then developed algorithms to prepare data to be solved by CPLEX (IBM, Armonk, NY) solution algorithms. A sample problem was prepared and solved to validate model and solution algorithms.

Results

Panel Width Optimization

Results of simulation experiments show that productivity and unit costs vary with varying panel width (see Figures 1 and 2). These results show that 11-entry advance always outperforms 13-entry advance with regards to productivity and costs. For the host mine, the optimal width, vis-à-vis productivity, is 17 entries using the 11-entry advance and three rooms on each side. Within the operating costs of the mine, unit costs decrease with increasing panel width over the simulated panel width range. Therefore, the optimal panel width when considering unit costs is 23 entries. However, further analysis shows that this result is sensitive to fixed costs. With high fixed costs, larger panels will provide lower unit costs since fixed costs far outweigh any gains in productivity.

As shown in the Figures 3 and 4, results are sensitive to the number of haulage units assigned to each continuous miner. The most gain is achieved when the number of cars increases from three to four. Hence, it is recommended that the host mine operate with four haulage units to optimize productivity. It should be noted that any addition of cars increases unit costs.

Optimal Room-and-pillar Mine Sequencing

Figure 5 shows results of the validation example with the solution respecting all constraints. Further work is required to properly evaluate the effect of risk and pillar recovery on the solution.
Conclusions

Panel Width Optimization

The research team successfully built a DES model for evaluating the effect of panel width on R&P mine productivity and unit costs. The model was validated for the host mine and used to evaluate the effect of panel width on the productivity and unit costs. Based on this work, the following conclusions can be made:

- For particular operating conditions, there exists an optimal panel width that maximizes productivity and/or minimizes unit costs, although each objective may result in a different answer.
- Fixed costs are a significant factor in determining optimal panel width based on unit costs.
- For the host mine, 11-entry advance with three rooms on each side is recommended based on productivity. While mining in this configuration, four haulage units should be assigned to each continuous miner.

![Figure 5: Validation results for mine panel sequencing optimization modeling.](image)

**Optimal Room-and-pillar Mine Sequencing**

The research team successfully built a MILP model for evaluating NPV while managing risk. The model and data preparation algorithms were validated by means of a sample problem that included 2,111 cuts in nine panels; 6,360 decision variables; and 14,822 constraints. Based on this work, the following conclusions can be made:

- Mining risk can be modeled using the MILP framework by introducing a risk factor and discounted penalty as well as assigning weights to control the significance of risk, relative to NPV, to the optimal mine sequence.
- The large number of block precedence constraints significantly affects solution time. The model presented here is able to obtain a feasible solution without the use of the block precedence, which significantly reduces computation time required to solve the problem.

Further work is required to explore the effect of risk and pillar extraction on the solution.
OBJECTIVES

At the project kickoff meeting, it was determined that the cooperating mine desired assistance in determining the optimal number of entries in a panel (panel width) to exploit their deposit. After discussions with all stakeholders, the scope of the original proposal was significantly modified to accommodate analyzing panel width as the main objective to be addressed with a reduced emphasis on the original objective of analyzing panel extraction sequencing while taking risk into account.

Panel Width Optimization

The objective of this part of the study was to evaluate the impact of panel width on unit cost and productivity of a room-and-pillar (R&P) operation using discrete event simulation (DES). This objective was achieved by (i) building a valid DES model that is representative of the mining system; and (ii) conducting simulation experiments with the validated model to determine the optimal panel width based on unit costs and productivity.

This part of the project was accomplished in the following five tasks:

- **Task 1 – Problem formulation**: This task included defining the problem and identifying all possible challenges. This involved discussions with all stakeholders to ensure the DES model reflected realistic scenarios and met the needs of both the mine and the Illinois coal industry.
- **Task 2 – System and simulation specification**: This task included defining the system (including its boundaries) and simulation model (including its assumptions and limitations). Part of the goal of the simulation exercise was to develop a DES model that is representative of the existing system, capable of evaluating the effect of panel width on mining cost and productivity, and capable of basic animation for verification and communicating model performance. The research team visited the mine and also performed time-and-motion studies at the mine. Model inputs and outputs were also defined in this task.
- **Task 3 – Modeling**: This task involved building a DES model of the system described in Task 2, above. The model was constructed using Arena® software (Rockwell Automation Inc., Milwaukee, WI).
- **Task 4 – Verification and validation**: To ensure the model behaves as intended and performs well when compared with the real system, verification and validation is performed in this task. The model is verified with animation and other techniques. Data from a production shift was used to validate the model.
- **Task 5 – Experimentation and analysis**: A set of experiments were designed to analyze the effect of varying panel width (number of entries) on unit mining cost and productivity. These experiments were designed to evaluate advancing a panel with either 11 or 13 entries, initially. Results were analyzed and recommendations made to the cooperating company.
Optimal Room-and-pillar Mine Sequencing

The goal of this portion of the study was to develop a mixed-integer linear programming (MILP) model that incorporates risk management in determining the optimal sequence of extraction for a R&P mine. The model maximizes net present value (NPV).

This part of the project was divided into four main tasks, as follows:

- **Task 1 – Modeling**: Formulation of the underground R&P mine sequencig problem as a mixed integer linear programming (MILP) problem.
- **Task 2 – Solution formulation**: Formulate algorithms that organize data for the MILP model as a set of vectors and matrices, which serve as input to CPLEX (IBM, Armonk, NY) that is used to solve the problem. Algorithms were also developed to post-process results obtained from the model.
- **Task 3 – Data acquisition**: Prepare R&P systems data to validate the model.
- **Task 4 – Validation**: Solve the problem prepared in Task 3 to show that algorithms are working as intended during modeling.

INTRODUCTION AND BACKGROUND

Panel Width Optimization

Room-and-pillar (R&P) mining is one of the oldest underground mining methods used for extracting flat and tabular deposits such as coal. The goal of R&P mine design is to extract the maximum possible amount of material while maintaining the strength and condition of the bearing rock. Design parameters in R&P mining depend on several factors, including production recovery, coal strength, depth of mining, and stability of the hanging wall (Farmer, 1992). A key aspect of R&P mine design is panel design, which depends on the strength and dimensions of the panel’s pillars as well as coal recovery and mine production requirements.

Paneling in R&P mining is done to divide the mine into different areas for ventilation and other aspects of mine operation. Barrier pillars are used to separate panels and prevent progressive collapse, in case a pillar in a panel fails. The size of barrier pillars increase as the width of the panel increases. Also, the number of barrier pillars needed to separate panels depends on the number of panels being planned. The size of a panel also affects the mining sequence with larger panels resulting in more complicated cut sequences and more tramming by continuous miners (CMs). It is, therefore, essential to note that panel width (number of entries) affects the overall cost and production recovery of the operation. An efficient mine design takes into account the optimum panel width that maximizes production and minimizes cost while ensuring bearing rock stability.

The traditional approach for determining panel width focuses primarily on geotechnical conditions, haulage distances (fleet requirements), and property boundaries (Dunrud, 1998; Loui and Sheorey, 2002; Zipf, 2001). However, it is important that panel width selection also takes into account maximizing overall productivity and minimizing
operating costs. The dimensions of a panel are determined by the number of rooms and pillars in the panel. The width of the panel is defined by the number of entries (development and production) excavated in the panel. Usually, greater emphasis is placed on panel designs when retreat mining is practiced. (Retreat mining is where entries and rooms are mined first and then pillars are mined afterwards.) The strategy employed in R&P panel development includes: (i) developing a base width (typically seven or more entries) on advance in increments (measured in crosscuts) that correspond with required belt and power moves, (ii) mining rooms on either side of the base width, and (iii) recovering pillars on retreat if that practice is allowed.

Although pillar recovery is not common in the Illinois Basin, there is a great need to design panels that are optimal. Recent advances in electric haulage units have spurred a move towards wider panels to take full advantage of their capabilities. However, the effect of wider panels on productivity and unit operating costs has not been investigated fully. Traditional design approaches do not facilitate optimization of unit mining costs and productivity as a function of panel width (Zipf, 2001; Loui and Sheorey, 2002). The objective of this study was to evaluate the impact of panel width (number of entries) on the cost and productivity of R&P operations, which are modeled with DES using Arena® software. DES is a computer-based simulation approach that models the behavior of complex systems as a discrete sequence of events. Often, this approach uses Monte Carlo simulation to handle uncertainty in the estimates.

**Optimal Room-and-pillar Mine Sequencing**

An important aspect of exploiting mineral resources is implementing a feasible and optimal mining sequence. R&P mining is one of the oldest underground mining methods and it is used primarily for mining flat and tabular deposits. In the USA, 90% of underground mines use the R&P method although the production sequence used in hard rock and coal mines vary as a result of different extraction techniques. Different R&P operations have different inherent risks associated with them. Production sequences in underground R&P mines depend primarily on the stability of the bearing rock mass as well as ventilation and production requirements (Tien, 2011). Risks associated with these factors make sequencing in R&P mines a challenge. The main challenges for modeling R&P mine sequencing include modeling several production cycle processes, managing mining risk (such as quality, production, and geotechnical risk (Alford et al., 2007)), and adhering to very strict sequencing requirements (Newman and Kuchta, 2010). Over the past five decades, several methods for modeling and solving R&P sequencing problems have been introduced with different operating objectives (Jawed, 1993; Sarin and West-Hansen, 2005).

Current research approaches do not account for multiple risk management in R&P mines. Common commercial software used by mining companies for production sequencing in R&P operations include Carlson, Geovia MineSched, Maptek, and XPAC. These software use heuristic methods that only find approximate solutions to production sequencing problems (i.e., they do not guarantee optimality). The focus of past research has been to develop schedules that optimize a specific objective, such as the amount of
material produced, operational costs, and material quality subject to model constraints (Jawed, 1993, Sarin and West-Hansen, 2005). The sequencing approach proposed in this study allows for controlling mining risk and its effect on optimal production sequences as well as maximizing NPV (multi-objective optimization). The approach is flexible, since it allows implementation of multiple mining risks in the same model. Unlike heuristic approaches, the proposed solution algorithm is capable of giving near optimal solutions (depending on termination criteria).

The objective of this study was to develop a MILP model that incorporates risk management into R&P mine sequencing while maximizing NPV. The R&P sequencing problem was formulated as a dual objective optimization problem subject to constraints such as precedence, production requirements, resource availability, quality, and reserve limits. Linear programming is a mathematical optimization approach used to find the optimizer of linear objective functions while satisfying a set of linear constraints. When decision variables are continuous (real) and discrete (integer), the problem is called a mixed integer linear program. There are four phases in solving decision making problems using MILP: (i) define the problem; (ii) construct the MILP model; (iii) solve the developed model; and (iv) validate the model and convert back to the decision situation (Chen et al., 2011). The advantage of MILP is its ability to model complex scheduling problems involving both continuous and discrete decision variables. It is a precise method that is capable of data manipulation. A limitation of using MILP is that mathematical modeling of large complex problems may be too abstract to comprehend and difficult to relate to reality. For problems with large constraints and integer variables, solution times can be significantly high and require considerable computing power. In fact, mine sequencing problems with their large precedence constraints are known to be particularly computationally expensive (Bienstock and Zuckerberg, 2010).

**EXPERIMENTAL PROCEDURES**

**Panel Width Optimization**

**Problem Formulation**

A DES model with variables that characterize the mining system was built using Arena®. The model is limited to the CM cutting sequence and haulage practices. The model predicts unit mining costs and productivity at different panel widths. It was validated with shift production data obtained from the host mine. The defined performance metric was that the relevant simulated output should be within 15% of actual values from the mine.

**System and Simulation Specifications**

The host mine for this study is located in southern Illinois. The mine produces approximately 7 million tons of coal per year from the Herrin No. 6 seam using R&P mining methods with a panel recovery rate of 54%. Eight Joy Model 14CM27 CMs (two for each panel) cut and load coal at up to 40 tons per minute with a maximum cutting height of 11.2 feet. Coal is hauled from CMs to feeder-breakers by 20-ton Joy Model
BH20 battery-powered haulage units. A feeder-breaker is located at the center of each production panel to transfer mined coal from haulage units to conveyor belts. It is moved forward as the panel advances in three-crosscut increments. The full width of the panel is mined in six-crosscut increments. The mine has experimented with different panel widths and mining sequences. Currently, 11 and 13 entries being advanced before mining rooms is the most common. Maximum and minimum panel widths are 21 and 11 entries, respectively. Each CM mines up to seven entries on one side of a panel.

The objective of the simulation is to develop a valid DES model that predicts unit mining costs and productivity and provides basic animation for verification. Input data used in the model were obtained from time studies done at the mine as described in Figures 6-11. Raw data were analyzed using the Chi-squared goodness-of-fit test to fit statistical distributions as shown in Table 1. Input data include loading and dumping times, payloads, and battery change data, which are sampled from the distribution. Model output includes production per shift, tons per hour, total operating costs including equipment costs, and the calculated cost per ton for a given panel width.

![Figure 6: Dumping time data.](image)

![Figure 7: Empty travel speed data.](image)

![Figure 8: Loaded travel speed data.](image)

![Figure 9: Loaded travel time data.](image)
Table 1: Input data distributions.

<table>
<thead>
<tr>
<th>Data</th>
<th>Distribution(s)</th>
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<tr>
<td>Payload (tons)</td>
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<tr>
<td>Empty travel speed (ft/sec)</td>
<td>6.11 + GAMM(0.327, 4.97)</td>
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<td>Loaded travel speed (ft/sec)</td>
<td>6 + GAMM(0.261, 4.24)</td>
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<td>Loading time (secs)</td>
<td>28 + ERLA(3.63, 3)</td>
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<tr>
<td>Dumping time (secs)</td>
<td>6 + GAMM(2.79, 5.36)</td>
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<tr>
<td>Battery change time (mins)</td>
<td>TRIA(5,7,10)</td>
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<td>Travel time between cuts (secs)</td>
<td>NORM(797, 87.7)</td>
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<tr>
<td>Spotting time (secs)</td>
<td>12.5 + GAMM(4.22, 2.11)</td>
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Model Formulation: CM and Haulage Logic

The DES modeling framework requires system entities, resources, and processes to be specified by the analyst. To initiate modeling, entities go through defined processes in a logical manner waiting for needed resources to become available at each process (i.e., resources are “busy” if they are being used by other entities) before they go through the process. The CM is modeled as a resource used for the loading process and can only load one haulage unit at a time. Loads of coal are modeled as entities with specific attributes (entity number, payload, and cut sequence). Battery-powered haulage units are modeled as guided transporters used for hauling loads (entities). A guided transporter is an Arena® specific modeling construct for material haulage. Transporters use entries and crosscuts as haulage routes, which are modeled to restrict traffic flow such that any point on a haulage route can only accommodate one haulage unit at a time since mine openings are not wide enough for them to pass each other. The feeder-breaker is also modeled as a stationary resource used for dumping loads (entities). The feeder-breaker and each cutting face are modeled as stations, which are points where entities are transferred by transporters. Haulage routes between stations are modeled as network links to capture varying haulage distances. Distances, in feet, for each network link are an input to the model. Figure 12 shows the logic used to model the system.
Figure 12: DES model logic.
Verification and Validation

An animation of the system was designed and used to verify that the model performs as intended. Shift production data from the host mine was used to validate the model. For validation, the simulation model predicted coal production (load count/shift) and shift duration, which was compared with data from a time-and-motion study conducted in one of the sections of the host mine where the panel was being advanced with 13 entries. The time-and-motion study collected data for 18 CM cuts completed over two non-consecutive shifts. During the first shift, 6.33 hours were spent making 11 cuts with the remaining time spent on conveyor belt and CM repairs. The second shift was spent entirely on production; however, data was collected for only the first half of the shift during which seven cuts were completed. For both shifts, coal was hauled by four haulage units with an average payload of 12 tons. In the validation experiment, 150 replications were conducted to obtain estimates of load count, total coal production, and duration of mining. The number of replications selected was such that the half-width is less than 1% of the load count. The cut sequence used in the validation experiment duplicated that used during the time-and-motion study.

Simulation Experiments and Analysis

Simulation experiments were designed to analyze the effect of panel width (number of entries) on mining cost and productivity. As a secondary goal, experiments were designed to determine whether to advance a panel with 11 or 13 entries before mining panel rooms. Both practices are used at the host mine. Hence, the experiment includes two factors:

- Number of entries for panel advance (11 or 13)
- Number of rooms (0, 1, ..., 5 rooms on each side of the panel)

Twelve experiments were conducted to cover all possible combinations. For each experiment, 150 replications were run for the analysis. Each replication was run until all cuts required to mine three crosscuts of advance were completed. The cut sequence was provided as an input based on mining practices at the host mine. Cuts in the 11 or 13 middle entries mined to advance the panel are made in a sequence similar to that shown in Figure 13. Cuts in rooms on either side of the panel are made in a sequence similar to that shown in Figure 14. The experiment evaluates a mining system with two CMs (one on each side of the section) and four haulage units assigned to each CM. The conveyor belt is located at the center of the panel. In order to evaluate the sensitivity of results to the number of haulage units assigned, all experiments were repeated with three and five haulage units assigned to each CM, in turn. This resulted in an additional 24 experiments.

Simulation output includes production data (e.g., load count and total production), duration of mining, and percentage of time the CM spends loading haulage units. With this information, the research team was able to compute other output such as total cost of mining and estimated unit costs using the following equation:
Unit costs ($/ton) = \frac{(n_{CM}C_{CM} + n_{H}C_{H})t_r + C_F}{\text{Total production}} \tag{1}

where \(n_{CM}\) and \(n_{H}\) are the number of CMs and haulage units, respectively; \(t_r\) is the simulation run duration; \(C_{CM}\) and \(C_{H}\) are hourly costs for the CM and haulage units, respectively; and \(C_{F}\) is fixed costs, which include labor and equipment for belt moves.

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**Figure 13:** Cut sequences for 11-entry panel advance.

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**Figure 14:** Cut sequences for two rooms on each side of panel.

**Optimal Room-and-pillar Mine Sequencing**

**MILP Modeling**

The object of this task was to model R&P coal mining operation using MILP that maximizes NPV while minimizing project risk. Decision variables are defined as \(x_{i,t}, y_{k,t}\), and \(z_{j,t}\), with block fraction \(i = 1, 2, \ldots, I\); pillar \(k = 1, 2, \ldots, K\); and panel \(j = 1, 2, \ldots, J\); mined in period \(t = 1, 2, \ldots, T\), respectively. The objective function shown in the following equation maximizes NPV while minimizing project risk:
In this equation, $p$ is price ($/ton); $r$ is overall recovery; $t_i$ and $t_k$ are tons in block $i$ and pillar $k$, respectively; $q_{it}$ and $q_{kt}$ are quality factors energy (BTU) and contaminant (ash) content of block $i$ and pillar $k$, respectively; $c_i$ is unit mining cost ($/ton); $d$ is the economic discount rate per period; $d_r$ is the discount rate of risk; $R_i$ and $R_k$ are risks that blocks and pillars mined in a period will have desired properties; $c_i$ and $c_k$ are discounted unit costs of deviation for each block and pillar, respectively; $x_{it}$ and $y_{kt}$ are binary variables indicating when a block and pillar are mined, respectively; and $w_e$ and $w_r$ are effective ratios for economic and risk factors.

**Constraints**

The model includes production constraints that ensure a feasible mining sequence. For this project, constraints included resource, precedence, mining rate, reserve, quality, and block-in-panel as defined in the following equations:

\[
R_i^t \leq \sum_{i,k} (r_{it} \cdot x_{it} + r_{kt} \cdot y_{kt}) \leq R_i^u \quad \forall t
\]

\[
N_i \sum_{j=1}^{i} x_{it} - \sum_{i \in O_j} \sum_{t=1}^{i} x_{it} \leq 0 \quad \forall i, \forall t
\]

\[
N_k \sum_{r=1}^{i} y_{kt} - \sum_{k \in O_i} \sum_{r=1}^{i} y_{kr} \leq 0 \quad \forall k, \forall t
\]

\[
N_j \sum_{t=1}^{i} z_{jt} - \sum_{j \in O_i} \sum_{t=1}^{i} z_{jt} \leq 0 \quad \forall j, \forall t
\]

\[
MR_{it}^l \leq \sum_{i,k} (t_{it} \cdot x_{it} + t_{kt} \cdot y_{kt}) \leq MR_{it}^u \quad \forall t
\]

\[
\sum_{t} x_{it} \leq 1, \quad \forall i
\]

\[
\sum_{t} y_{kt} \leq 1, \quad \forall k
\]

\[
\sum_{t} z_{jt} \leq 1, \quad \forall j
\]
Equation 3 constrains the model from exceeding available resource capacity \( R_t^b \) or under-utilizing available resources \( R_t^l \) in a particular period, where \( r_{it} \) and \( r_{kt} \) are the amount of resource capacity required to mine block \( i \) and pillar \( j \) in period \( t \), respectively. This results in \( T \) (number of scheduling periods) constraints for each modeled resource. Mining resources may include production and development equipment, labor, and other auxiliary elements required to extract the material being mined. In the validation example for this project, CM time is the only relevant resource, for simplicity.

Equations 4(a-c) constrain block and pillar mining precedence, which is the single most significant contributor to the problem’s complexity (Bienstock and Zuckerberg, 2010). The equation results in \((I+K+J) \cdot T\) constraints ensuring that a block, pillar, or panel cannot be mined until the set of blocks, block and panels, or panels that restrict it are all mined first. This constraint allows for practical mining of blocks, pillars, and panels. For each block, pillar, and panel, a set \( \{O_i, O_k, O_j\} \) of blocks, blocks and pillars, and panels, respectively, are defined to be mined prior to extracting block \( i \), pillar \( k \), and panel \( j \). The research team explored ways to solve the coal R&P sequencing problem without these constraints in order to save significant computational time.

Equation 5 ensures that the total tonnage mined in each period is within targets \( MR_t^b \) and \( MR_t^l \) established for mine production. This results in \( T \) constraints, one for each period.

Equations 6(a-c) ensures that the amount of material mined at the end of the time period is less than or equal to the available reserve tonnage. This results in \((I+K+J)\) constraints.

Equation 7 ensures that the solution meets quality requirements. For each quality property of interest, there is a separate equation. In coal mining, quality properties of interest include energy, sulfur, and mercury content. Each set will result in \( T \) constraints to ensure that average constituents (BTU and contaminants) mined in a period are within the desired range \( (q_{i}^{l} \) and \( q_{i}^{u} \)) for that period, where \( q_{i}^{l} \) and \( q_{i}^{u} \) are averages for each block or pillar, respectively. This constraint forms a basis for blending and quality control.

Equation 8 ensures that once a panel is scheduled for mining, all blocks in the panel are mined in the same period. This constraint is used without block and pillar precedence constraints (Equations 4a and 4b) and for it to work properly, decision variables have to be binary. This results in \((I+K) \cdot T\) constraints, compared to \((I+K+J) \cdot T\) block-pillar precedence constraints, which is significant. Preliminary trials with the model required over eight hours to solve on a 64-bit computer with twin 2.5GHz processors and 32 GB RAM. The same problem took less than one minute to solve with this approach.
Solution Formulation

Equations 2-8 are solved using the CPLEX solver (IBM, Armonk, NY) through the CPLEX API for Matlab. A generalized mixed integer linear program is formulated mathematically in the form of Equation 9. Input required by CPLEX are the cost (or benefit) coefficient vector \( c \), generated from the objective function; the equality constraint matrix \( (A_{eq}) \); the inequality constraint matrix \( (A) \); and limits (right hand-side of constraint equations) of inequality and equality constraints \( (b \text{ and } b_{eq}) \), respectively. CPLEX solver also requires other constraints on decision variables (integer, binary, etc.).

\[
\min \ z = c \mathbf{x} \\
\text{subject to:} \\
A_{eq} \mathbf{x} = b_{eq} \\
A \mathbf{x} \leq b
\]

\[
c^x_i = w_i \sum_{r,t} \left( \frac{p_{r,t} I_r q_{i,r} - c_{r,i}}{(1+d)^t} \right) - w_i \sum_{r,t} \frac{R_{r,t} c_{r,i}}{(1+d_r)^t}
\]

\[
c^y_k = \sum_{r,t} \left( \frac{p_{r,t} I_r q_{k,r} - c_{r,k}}{(1+d)^t} \right) - w_i \sum_{r,t} \frac{R_{r,t} c_{r,k}}{(1+d_r)^t}
\]

From Equation 2, elements of \( c \) are present values of blocks less the risk penalty associated with mining these blocks in a particular period. Hence, the length of \( c \) is the number of decision variables. The developed solution algorithm allows the user to provide a set of blocks and time periods, block properties (BTUs, tonnages, etc.), discount rates, risks, and other input, which the algorithm uses to generate the vector \( c \) as shown above. The solution algorithm also creates constraint matrices using information provided by the user. Equation 8 results in an equality constraint used to formulate \( A_{eq} \) and \( b_{eq} \) in Equation 9. All other constraints are inequality constraints. For each constraint,
the solution algorithm needs to formulate a matrix (which becomes part of \( A \) in Equation 9) and a right-hand side (RHS) vector, which becomes part of \( b \) in Equation 9.

The Matlab program is set up so that the user can provide the amount of each resource required to mine each block and the total resource available in each period. Thus, resources can be controlled for the life of the mine to maintain a feasible mining schedule and efficient use of resources. To formulate the mining rate matrix, the algorithm requires the tonnage of material in each block and the production demand for each period. The program requires the user to provide information that the algorithm uses to select the block and pillars in a particular panel. The program also requires the user to provide the average quality/constituent of each block and limits on each constituent for each period. Thus, the user can set upper and/or lower limits on BTU content, sulfur content, etc. for each period. The precedence constraint requires users to provide indices for each block, pillar, and panel. For each block, the set of blocks that precede it are in the same panels. A set of indices are used to describe each panel such that panels that precede another panel can be mined first. All inequality constraint matrices and RHS vectors are concatenated into a single matrix and a single vector. Along with the cost coefficient vector, these serve as input for the CPLEX solution.

**Data Acquisition**

Data used to validate the MILP model is numeric based on a typical R&P coal mining system. The validation problem did not include pillar extraction since the host mine does not practice retreat mining, but it did included 2,111 blocks with each block representing a cut assumed to contain 250 tons of coal. The problem is solved for three periods involving 6,360 decision variables and 14,822 constraints including 6,333 equality constraints. Other input data need to validate the MILP model is shown in Table 2. The problem evaluated risk associated with BTU estimates in cuts (i.e., what is the risk that BTU in a cut is lower than estimated). Risk and NPV in the problem are considered of equal importance.

**Table 2: Input data.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of mining</td>
<td>$2/ton</td>
<td>CM capacity per period</td>
<td>356.57 hrs.</td>
</tr>
<tr>
<td>Coal price</td>
<td>$65.55/ton</td>
<td>Production target lower limit</td>
<td>(1.43 \times 10^5 \text{ tons})</td>
</tr>
<tr>
<td>(assuming bituminous coal)</td>
<td></td>
<td>Production target upper limit</td>
<td>(1.94 \times 10^6 \text{ tons})</td>
</tr>
<tr>
<td>Cost of BTU risk</td>
<td>$7.58/ton</td>
<td>Recovery</td>
<td>100%</td>
</tr>
<tr>
<td>Discount rate (economic)</td>
<td>8%</td>
<td>BTU target</td>
<td>13,000 BTU</td>
</tr>
<tr>
<td>Discount rate (risk)</td>
<td>5%</td>
<td>Block BTU mean</td>
<td>13,182 BTU</td>
</tr>
<tr>
<td>Block tonnage</td>
<td>250 tons</td>
<td>Block BTU mean</td>
<td>13,182 BTU</td>
</tr>
<tr>
<td>CM capacity per block</td>
<td>26.62 min.</td>
<td>Block BTU std. dev.</td>
<td>481 BTU</td>
</tr>
</tbody>
</table>
**Validation**

The validation problem included nine panels. Panel precedence sets used in the validation example are shown in Table 3. Block precedence sets are not considered in the solution and are too numerous to include in this report. They were used only to get an idea of how much time is saved by the model.

**Table 3**: Panel precedence sets.

<table>
<thead>
<tr>
<th>Panel No.</th>
<th>Precedence Set</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>{ }</td>
</tr>
<tr>
<td>2</td>
<td>{1}</td>
</tr>
<tr>
<td>3</td>
<td>{1}</td>
</tr>
<tr>
<td>4</td>
<td>{1,7}</td>
</tr>
<tr>
<td>5</td>
<td>{1,7}</td>
</tr>
<tr>
<td>6</td>
<td>{1,7}</td>
</tr>
<tr>
<td>7</td>
<td>{1}</td>
</tr>
<tr>
<td>8</td>
<td>{1,7}</td>
</tr>
<tr>
<td>9</td>
<td>{1,7}</td>
</tr>
</tbody>
</table>

**RESULTS AND DISCUSSION**

**Panel Width Optimization**

**Validation**

Table 4 shows results of validation experiments. Mining duration is longer (30 minutes) for the model, which also loads 24 more cars than was observed during time-and-motion studies. Key performance measures were loads and mining duration for 11 cuts, which are within 11% and 8% of actual values, respectively. This meets the 15% criteria established for validity. Thus, the model was deemed valid.

**Table 4**: Results of validation experiment.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Actual</th>
<th>Simulated</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration of mining (hours)</td>
<td>6.33</td>
<td>6.83</td>
<td>8%</td>
</tr>
<tr>
<td>Production (tons)</td>
<td>2,448</td>
<td>2,748</td>
<td>12%</td>
</tr>
<tr>
<td>Number of haulage unit loads</td>
<td>204</td>
<td>226</td>
<td>11%</td>
</tr>
<tr>
<td>Half-width of duration (hours)</td>
<td>--</td>
<td>0.012</td>
<td>--</td>
</tr>
</tbody>
</table>

**Effect of Advance Entries and Number of Rooms**

Figures 15-22 show results for the first set of simulation experiments. These experiments were to show the effect of panel width (number of entries) on productivity and unit costs.
In all simulations, each CM was assigned four haulage units. Figures 15 and 16 show that total production and duration of mining increase with increasing number of entries, which is as expected. Figure 17 and 18 show that the percentage of production time the CM spends loading haulage units (CM utilization rate) is optimal with at least three rooms on each side for the 11-entry system and at least one room on each side for the 13-entry system. This indicates that there is excess CM capacity in the system with less than the optimal number of entries; however, expanding the panel any more than this optimal number of entries results in inadequate haulage unit capacity and under-utilization of the CM. This is confirmed by haulage unit cycle times shown in Figures 19 and 20. Initial expansion of the panel reduces haulage unit cycle time (minimizes waiting time); however, expansion of the panel beyond optimal width increases haulage unit cycle times because the system is now constrained by the number of cars rather than CM capacity.

![Figure 15: Total production.](image1)

![Figure 16: Duration of mining.](image2)

![Figure 17: CM utilization (left side).](image3)

![Figure 18: CM utilization (right side).](image4)
These trends (cycle times and CM loading times) directly result in the observed trend in productivity (Figure 1, shown again in Figure 21). Panel widths of 17 and 19 entries result in maximum productivity when advancing with 11 and 13 entries, respectively. Unit cost results (Figure 2, shown again in Figure 22) do not indicate the same optimum width, as would be expected. This is due to the effect of fixed costs, which make larger panels more cost effective even with sub-optimal productivity. In Figure 22, unit costs are estimated using Equation 1 with hourly costs of haulage units and CMs estimated at $79.13 and $122.40, respectively (Infomine, 2013). Fixed costs for belt moves are estimated at $81,050.
The following observations are made based on modeling the effect of number of entries:

- 11-entry systems outperform 13-entry systems under similar conditions (cut sequences and equipment);
- Haulage unit cycle times correlate very well to productivity and CM time spent loading;
- There appears to be an optimal panel width for a given number of haulage units based on productivity analysis; and
- Unit costs decrease with increasing number of entries due to the effect of fixed costs.

**Effect of Number of Cars**

Figures 23 and 24 show the sensitivity of productivity results to number of haulage units. It can be observed that with the addition of each car, productivity increases; however, the increase in productivity when the number of cars increases from three to four is much more significant than the increase when the number of cars increases from four to five. Also, the number of cars can affect optimal panel width. For example, Figure 23 shows that the optimal panel width with three cars assigned to each CM is 13 entries whereas with four cars, the optimal panel width is 17 entries. This is because the number of cars affects the width at which the system becomes limited by haulage unit capacity.

![Figure 23](image1)

**Figure 23:** Effect of number of cars on productivity for 11-entry system.

![Figure 24](image2)

**Figure 24:** Effect of number of cars on productivity for 13-entry system.

Figures 25 and 26 show the sensitivity of unit cost results to number of haulage units. With each additional car, unit costs increase for both 11- and 13-entry systems.
The following observations are made based on modeling the effect of number of cars:

- Results are sensitive to the number of haulage units such that:
  - Productivity increases with additional cars, and
  - The optimal number of entries changes with varying number of cars; and
- The increase in costs outpaces the increase in productivity with each additional car leading to higher unit costs.

**Effect of Fixed Costs**

From Equation 1, if fixed costs are negligible, the unit cost curve should be the inverse of the productivity relationship; however, Figures 21 and 22 do not show that to be the case suggesting that fixed costs significantly affect the unit cost relationship. Figure 27 shows the sensitivity of the unit cost relationship to fixed costs displaying results for the 11-entry system with four cars (the same relationship shown in Figure 22). Figure 27 indicates that the unit cost relationship will indeed show an optimal at 17 entries if fixed costs are low (≤$1,000). Fixed costs as low as $2,000 are more than adequate to compensate for any drop in productivity from under-resourced CMs. For high fixed costs (≥$2,000), the unit cost for mining larger panels will be lower, even though productivity will be sub-optimal after the panel width exceeds the optimal for productivity. From a cost perspective, larger panels are advantageous because of fixed costs included in belt and power moves.
Figure 27: Effect of fixed costs on unit cost relationships.

Optimal Room-and-pillar Mine Sequencing

Figures 28 and 29 show results of the validation problem. As can be seen from these figures and Tables 2 and 3, the solution respects all constraints. These results were obtained in less than a minute compared to over eight hours for the same problem with block precedence constraints specified.

Figure 28: Sequence optimization results.
Further work is necessary to solve real life problems for R&P coal mines. More realistic periods and panels should be solved before any judgment can be made on the usefulness of this model. The expectation is that as the number of panels and periods increase, the problem will get more complex and the strength of this modeling approach over using block precedence can be better evaluated. It may even be that a combination of block precedence and panel-block relationships are required to solve the problem realistically.

Due to the revised scope, the research team was unable to explore relationships between risk and profitability, which was the main goal of the original proposal. The team is confident that risk has been successfully incorporated into the problem; however, without further trials, it is not possible to make any conclusions on the effect of risk on the complexity of the problem or the nature of the solution. Similarly, the team was unable to explore any advantages of incorporating pillar extraction into the problem. Hence, no conclusions can be drawn on the benefit of incorporating pillar extraction into the optimization problem as compared to conventional practice.

CONCLUSIONS AND RECOMMENDATIONS

Panel Width Optimization

This research effort has successfully built a discrete event simulator capable of evaluating the effect of panel width (number of entries) on R&P mine productivity and unit costs. The simulator has successfully been validated and used to evaluate the effect of panel width on productivity and unit costs at the host mine. Based on results from this modeling effort, the following general conclusions can be made:

- For particular operating conditions (equipment, cut sequence etc.), there exist an optimal panel width that maximizes productivity.
For particular operating conditions, an optimal panel width exists that minimizes unit costs, only if fixed costs are negligible. For any significant level of fixed costs, larger panels will always result in lower unit costs.

Fixed costs are a significant factor in deciding the optimal panel width based on unit costs.

For the host mine in particular, the following conclusions can be drawn:

- The 11-entry system is better than the 13-entry system for panel advance before mining rooms. This is a function of the cut sequences adopted at this mine. In particular, the practice of moving the belt after advancing three crosscuts ensures that haul distances to rooms are reasonable.
- The optimal panel width under simulated conditions is 17 entries (11 entries on advance and three rooms on each side).
- Four haulage units should be assigned to each CM in this mine.

**Optimal Room-and-pillar Mine Sequencing**

Revising the scope of the original project affected the ability of the research team to fully develop the proposed optimal R&P mine sequencing algorithm. Models were successfully formulated that incorporate pillar recovery and risk into R&P mine sequencing. From the research thus far, the following conclusions can be drawn:

- Mining risk can be modeled using a mixed integer programming framework by introducing a risk factor and discounted penalty that control production sequencing for risk. The significance of risk on the optimal mine sequence is controlled by assigning weight ratios to the risk model in the objective function.
- The mine sequencing model has been validated with a “small” problem that includes 2,111 cuts in nine panels, 6,360 decision variables, and 14,822 constraints.
- As expected, the large number of block precedence constraints significantly affects solution time. The model presented here is able to obtain a feasible solution without the use of block precedence. This significantly reduces computational time required to solve the problem.

Further work is required to explore the effect of risk and pillar extraction on solution times and the solution itself. This will require preparing realistic field data to test the model and solution algorithms.
REFERENCES


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